SEG5010 presentation

GRAPH COMPRESSION AND SUMMARIZATION

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- Most of the slides are borrowed from the authors' original presentation.
  - http://www.cs.umd.edu/~saket/pubs/sigmod2008.ppt
  - http://videolectures.net/kdd09_kumar_ocsn/

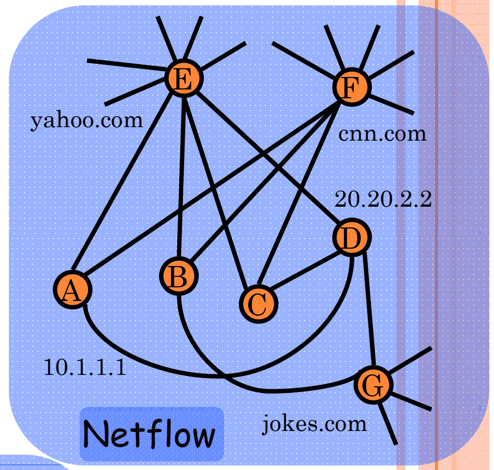
Graph Summarization with Bounded Error

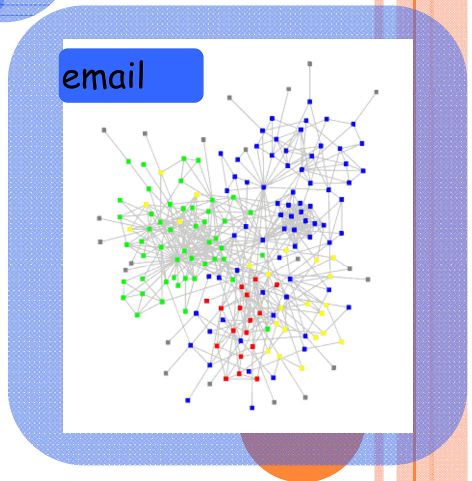
- Saket Navlakha (UMCP)
- Rajeev Rastogi (Yahoo! Labs, India)
- Nisheeth Shrivastava (Bell Labs India)

LARGE GRAPHS

- Many interactions can be represented as graphs
  - Webgraphs: search engine, etc.
  - Netflow graphs (which IPs talk to each other): traffic patterns, security, worm attacks
  - Social (friendship) networks: mine user communities, viral marketing
  - Email exchanges: security. virus spread, spam detection
  - Market basket data: customer profiles, targeted advertizing

- Need to compress, understand
  - Webgraph ~ 50 billion edges; social networks ~ few million, growing quickly
  - Compression reduces size to one-tenth (webgraphs)





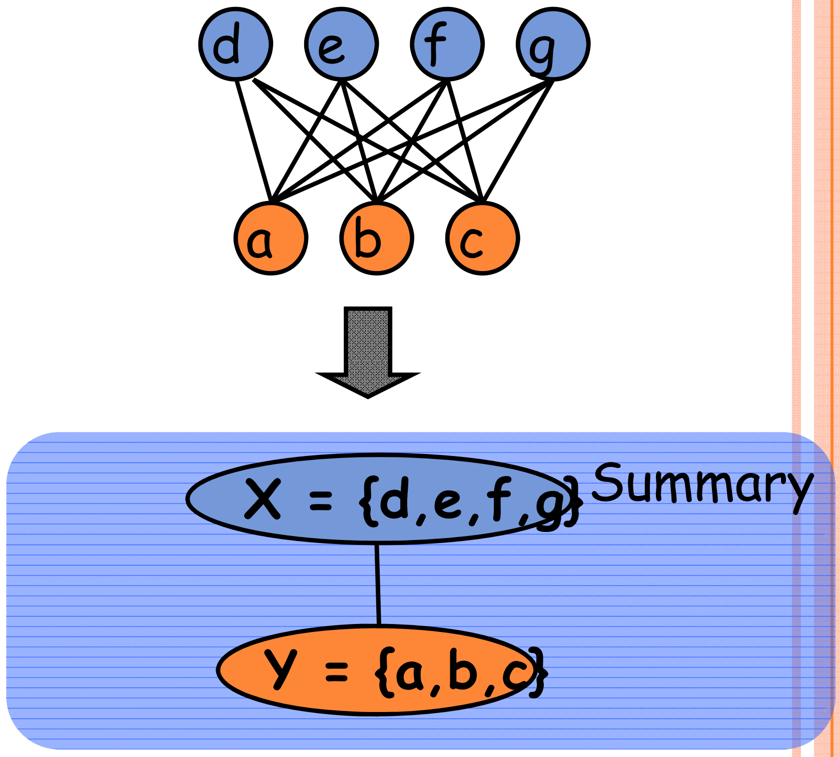
Our Approach

- Graph Compression (reference encoding)
  - Not applicable to all graphs: use urls, node labels for compression
  - Resulting structure is hard to visualize/interpret
- Graph Clustering
  - Nice summary, works for generic graphs
  - No compression: needs the same memory to store the graph itself

- Our MDL-based representation R = (S,C)
  - *S is a high-level summary graph:* compact, highlights dominant trends, easy to visualize
  - *C is a set of edge corrections:* help in reconstructing the graph
  - Compression based on MDL principle: minimize cost of S+C information-theoretic approach; parameter less; applicable to any graph
  - Novel Approximate Representation: reconstructs graph with bounded error (ϵ); results in better compression

How do we compress?

- Compression possible (S)
  - Many nodes with similar neighborhoods
    - Communities in social networks; link-copying in webpages
  - Collapse such nodes into *supernodes* (clusters) and the edges into *superedges*
    - Bipartite subgraph to two supernodes and a superedge
    - Clique to supernode with a "self-edge"



HOW DO WE COMPRESS?

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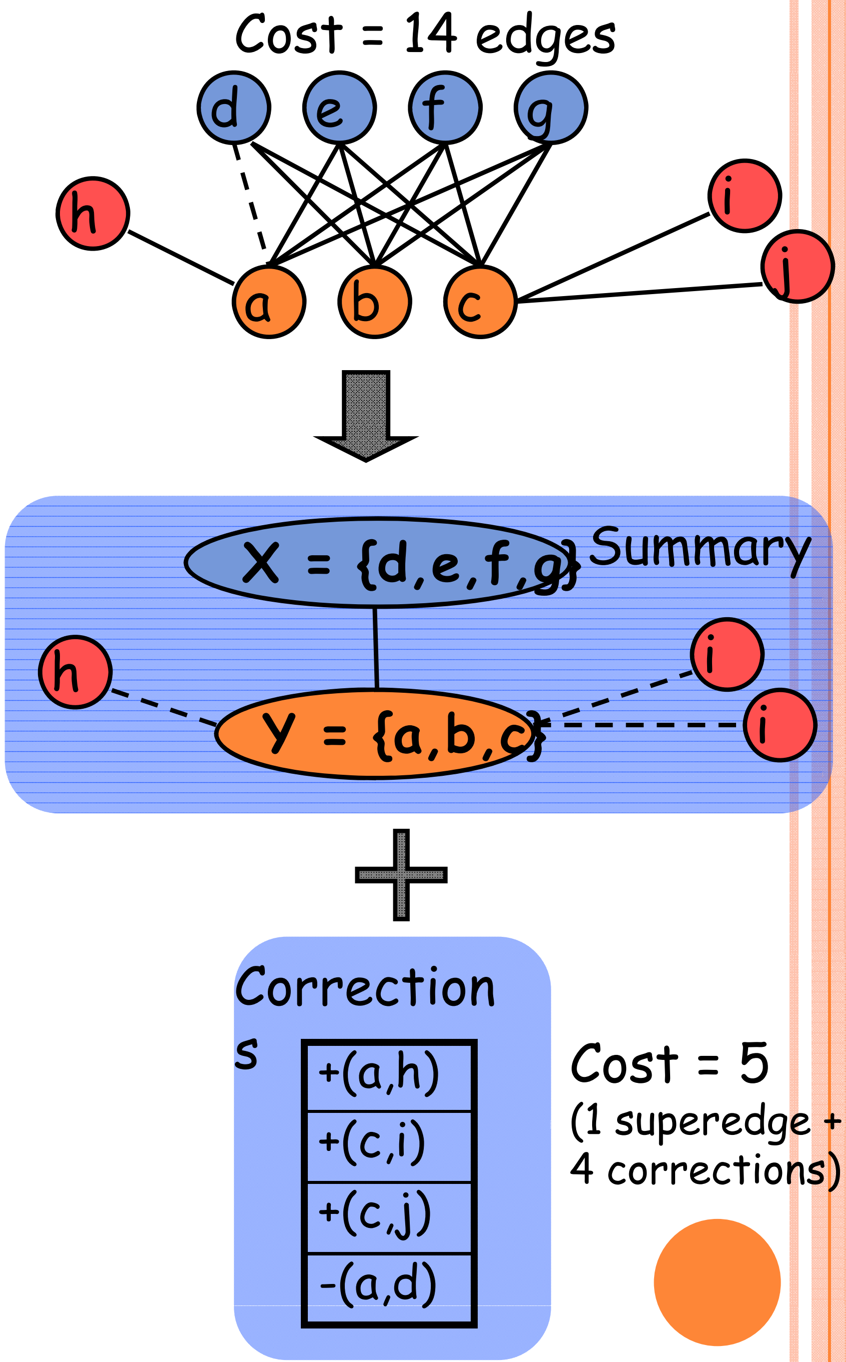
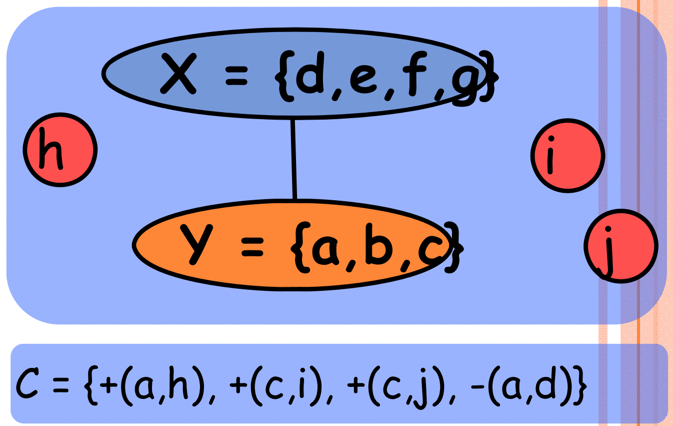
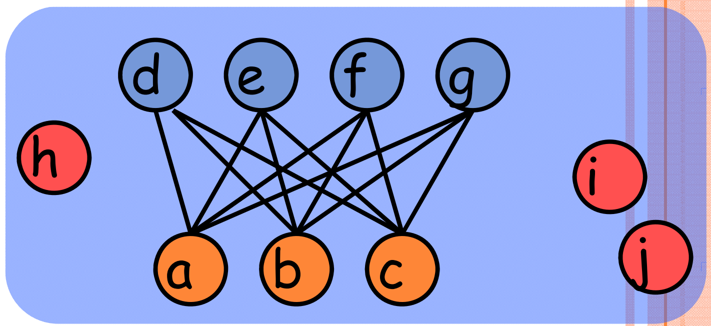
  - Collapse such nodes into *supernodes* (clusters) and the edges into *superedges*
    - Bipartite subgraph to two supernodes and a superedge
    - Clique to supernode with a "self-edge"
- Need to correct mistakes (C)
  - Most superedges are not *complete*
    - Nodes don't have exact same neighbors: friends in social networks
  - Remember edge-corrections
    - Edges not present in superedges (-ve corrections)
    - Extra edges not counted in superedges (+ve corrections)
- Minimize overall storage cost = S+C



Representation Structure R=(S,C)



- Summary $S(V_S, E_S)$
  - Each supernode v represents a set of nodes A_v
  - Each superedge (u,v) represents all pair of edges $\pi_{uv} = A_u \times A_v$
- Corrections C: {(a,b); a and b are nodes of G}
- Supernodes are key, superedges/corrections easy
  - A_{uv} actual edges of G between A_u and A_v
  - Cost with (u,v) = $1 + |\pi_{uv} - E_{uv}|$
  - Cost without (u,v) = $|E_{uv}|$
  - Choose the minimum, decides whether edge (u,v) is in S



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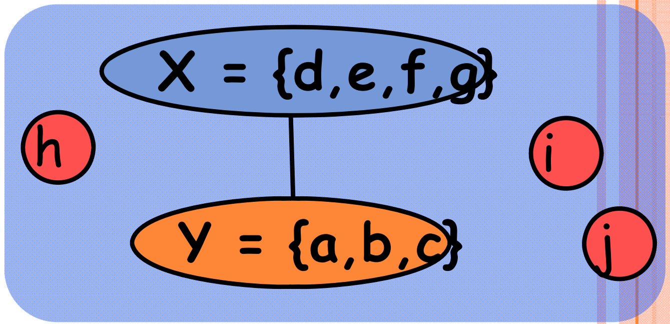

C = {+(a,h), +(c,i), +(c,j), -(a,d)}

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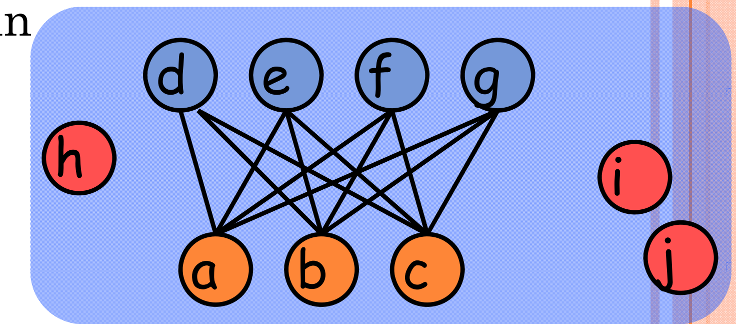

- Reconstructing the graph from R
  - For all superedges (u,v) in S, insert all pair of edges π_{uv}
  - For all +ve corrections +(a,b), insert edge (a,b)
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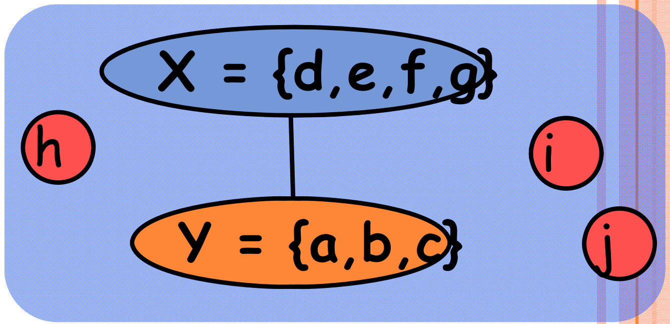

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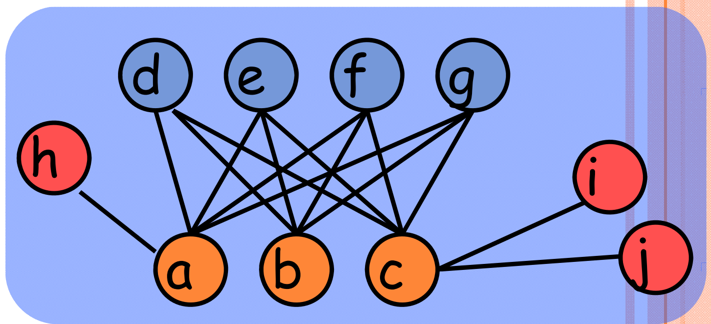

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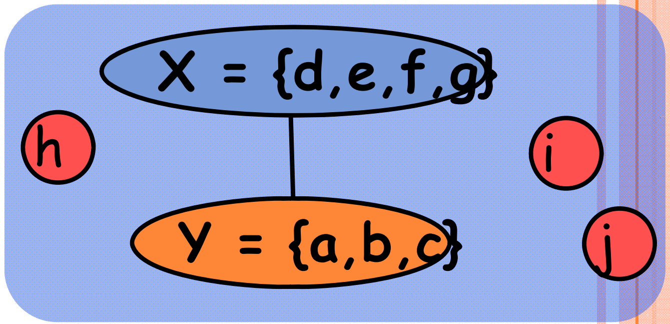

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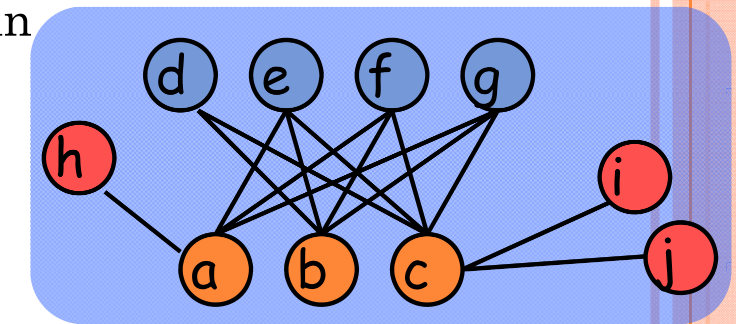

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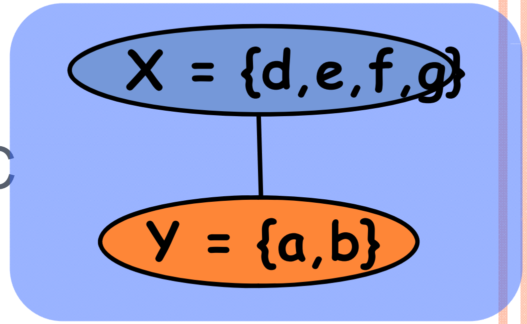
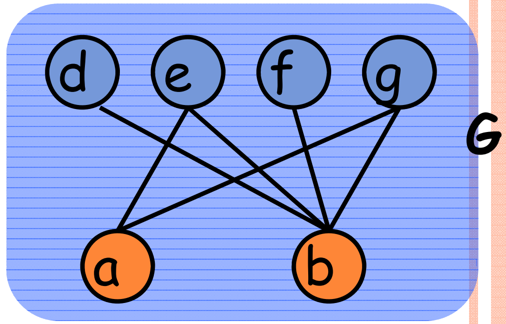
Approximate Representation R_ϵ

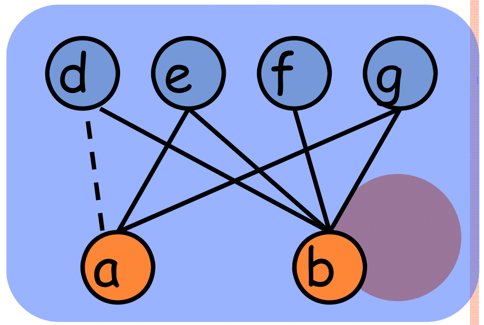
- Approximate representation
  - Recreating the input graph *exactly* is not always necessary
  - Reasonable approximation enough: to compute communities, anomalous traffic patterns, etc.
  - Use approximation leeway to get further cost reduction
- Generic Neighbor Query
  - Given node v, find its neighbors N_v in G
  - Apx-nbr set N'_v estimates N_v with ϵ-accuracy
  - Bounded error: error(v) = $|N'_v - N_v| + |N_v - N'_v| < \epsilon |N_v|$
  - Number of neighbors added or deleted is at most ϵ-fraction of the true neighbors
- Intuition for computing R_ϵ
  - If correction (a,d) is deleted, it adds error for both a and d
  - From exact representation R for G, remove (maximum) corrections s.t. ϵ-error guarantees still hold



C = {-(a,d), -(a,f)}

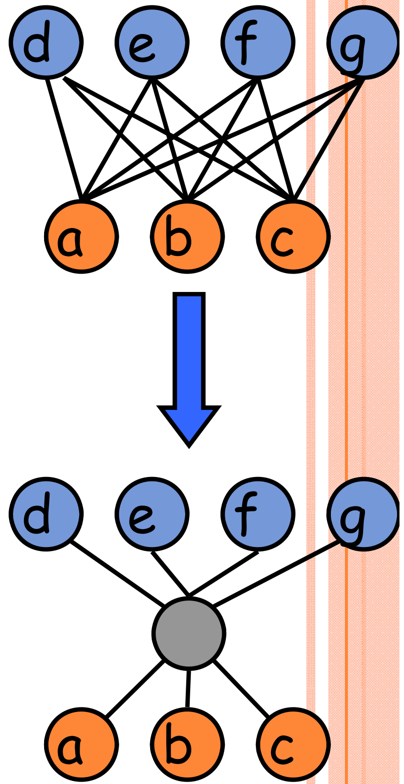


For ϵ=.5, we can remove one correction of a



COMPARISON WITH EXISTING TECHNIQUES

- **Webgraph compression** [Adler-DCC-01]
  - Use nodes sorted by urls: not applicable to other graphs
  - More focus on bitwise compression: represent sequence of neighbors (ids) using smallest bits
- **Clique stripping** [Feder-pods-99]
  - Collapses edges of complete bi-partite subgraph into single cluster
  - Only compresses very large, complete bi-cliques
- **Representing webgraphs** [Raghavan-icde-03]
  - Represent webgraphs as SNodes, Sedges
  - Use urls of nodes for compression (not applicable for other graphs)
  - No concept of approximate representation



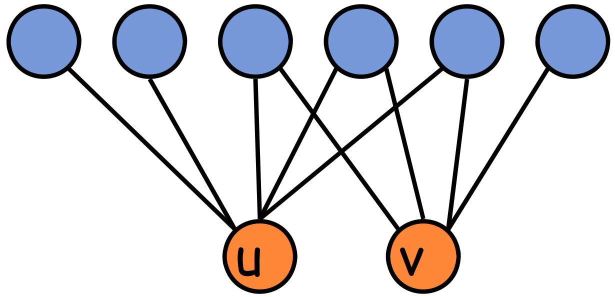
Outline

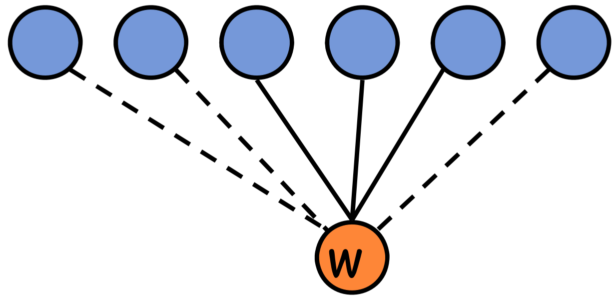
- Compressed graph
  - MDL representation R=(S,C); ϵ-representation
- Computing R
  - GREEDY, RANDOMIZED
- Computing R_ϵ
  - APX-MDL, APX-GREEDY
- Experimental results
- Conclusions and future work

GREEDY

- Cost of merging supernodes u and v into single supernode w
  - Recall: cost of a superedge (u,x):
    $c(u,x) = \min\{|\pi_{vx} - A_{vx}|+1,\ |A_{vx}|\}$
  - c_u = sum of costs of all its edges = $\Sigma_x\, c(u,x)$
  - **$s(u,v) = (c_u + c_v - c_w)/(c_u + c_v)$**

- Main idea: recursive bottom-up merging of supernodes
  - If s(u,v) > 0, merging u and v reduces the cost of reduction
  - Normalize the cost: remove bias towards high degree nodes
  - Making supernodes is the key: superedges and corrections can be computed later

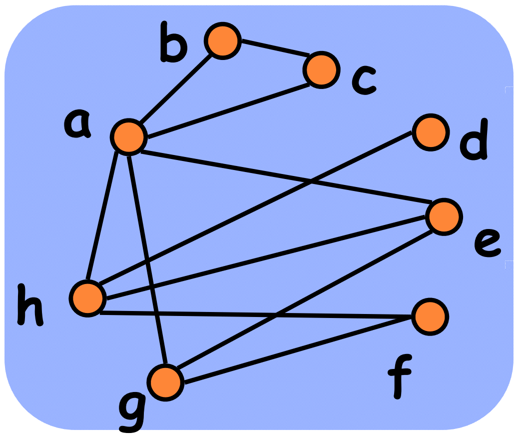




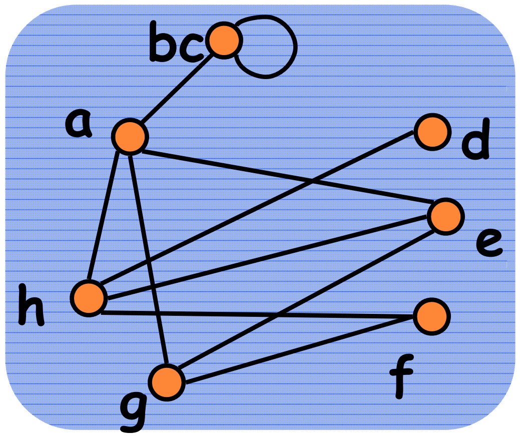
c_u = 5; c_v =4
c_w = 6 (3 edges, 3 corrections
s(u,v) = 3/9

GREEDY

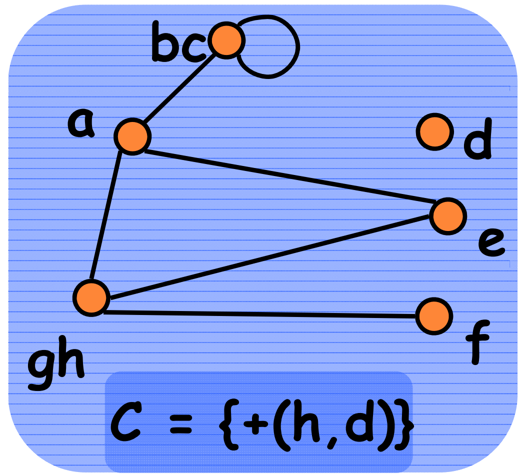
- Recall: $s(u,v) = (c_u + c_v - c_w)/(c_u + c_v)$
- GREEDY algorithm
  - Start with S=G
  - At every step, pick the pair with max s(.) value, merge them
  - If no pair has positive s(.) value, stop



s(b,c)=.5
[$c_b = 2$; $c_c=2$; $c_{bc}=2$]

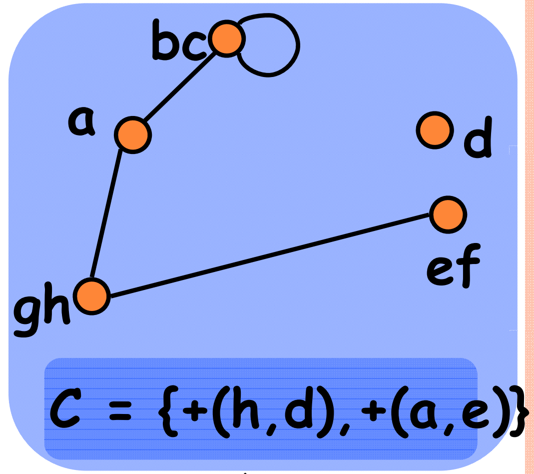


s(g,h)=3/7
[$c_g = 3$; $c_h=4$; $c_{gh}=4$]



s(e,f)=1/3
[$c_e = 2$; $c_f=1$; $c_{ef}=2$]

Cost reduction: 11 to 6

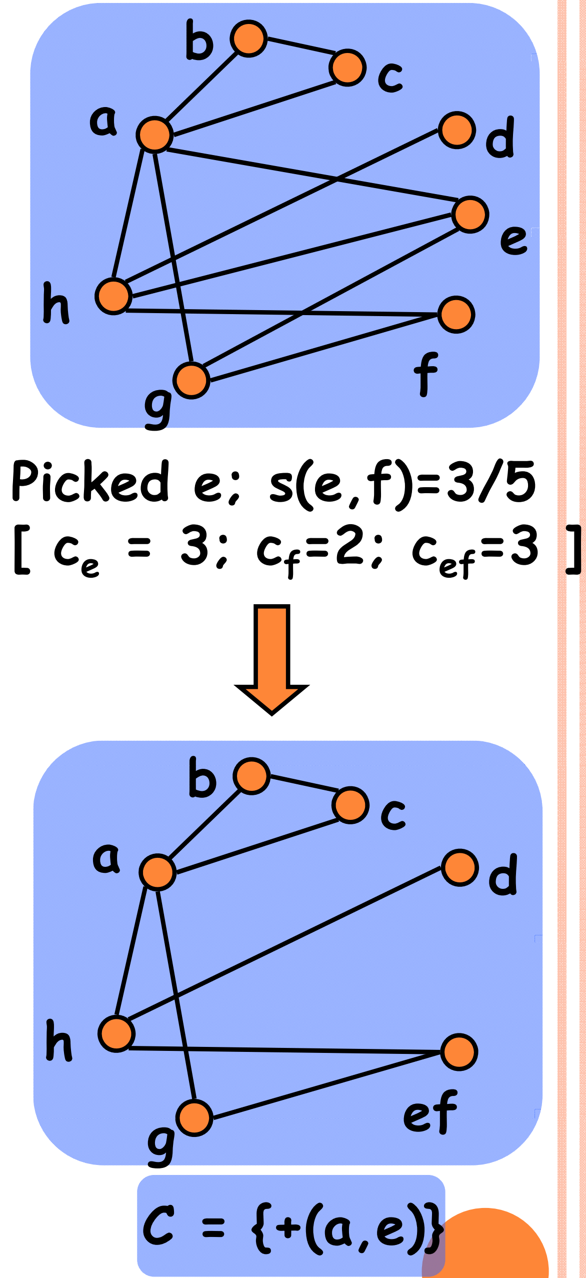


RANDOMIZED

- GREEDY is slow
  - Need to find the pair with (globally) max s(.) value
  - Need to process all pair of nodes at a distance of 2-hops
  - Every merge changes costs of all pairs containing N_w
- Main idea: light weight randomized procedure
  - Instead of choosing the globally best pair,
  - Choose (randomly) a node u
  - Merge the best pair containing u

RANDOMIZED

- Randomized algorithm
  - Unfinished set $U=V_G$
  - At every step, randomly pick a node u from U
  - Find the node v with max s(u,v) value
  - If s(u,v) > 0, then merge u and v into w, put w in U
  - Else remove u from U
  - Repeat till U is not empty



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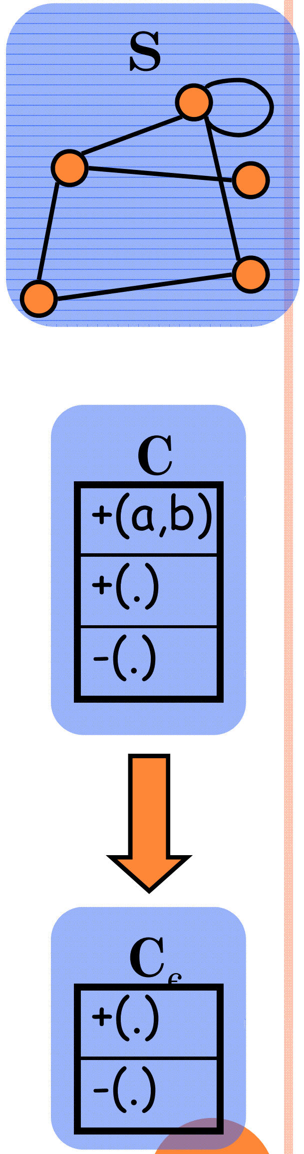
Computing approx representation

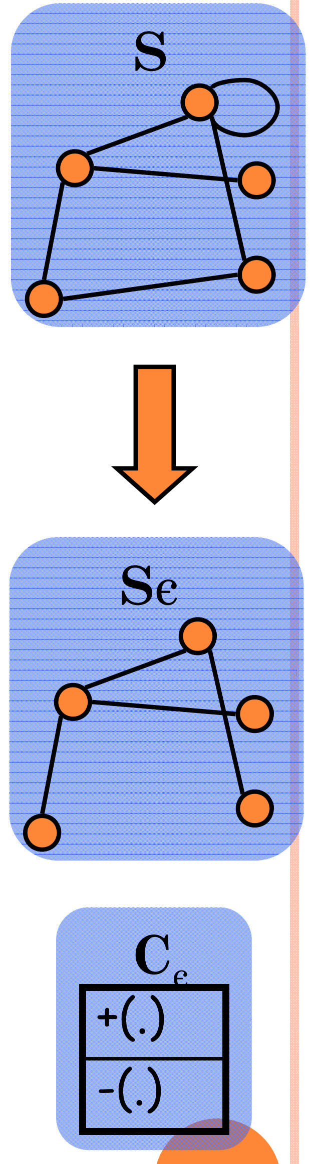
- Reducing size of corrections
  - *Correction graph* H: For every (+ve or –ve) correction (a,b) in C, add edge (a,b) to H
  - Removing (a,b) reduces size of C, but adds error of 1 to a and b
  - Recall bounded error: error(v) = $|N'_v - N_v| + |N_v - N'_v| < \epsilon\, |N_v|$
  - Implies in H, we can remove up to $b_v = \epsilon\, |N_v|$ edges incident on v
  - Maximum cost reduction: remove subset M of E_H of max size s. t. M has at most b_v edges incident on v

- Same as the b-matching problem
  - Find the matching M\subset E_G s.t. at most b_v edges incident on v are in M
  - For all $b_v = 1$, traditional matching problem
  - Solvable in time $O(mn^2)$ [Gabow-STOC-83] (for graph with n nodes and m edges)



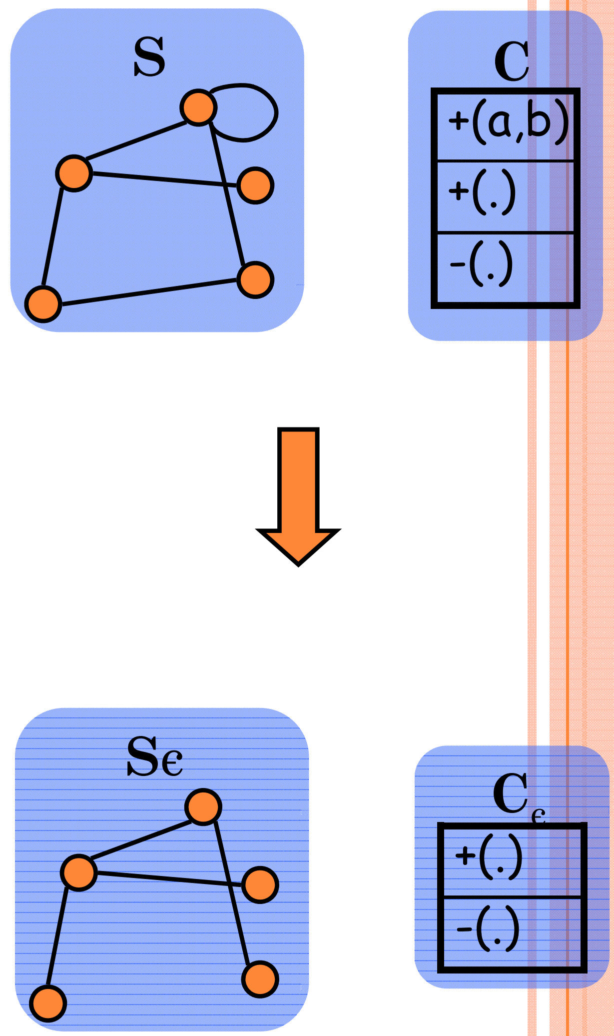
COMPUTING APPROX REPRESENTATION

- Reducing size of summary
  - Removing superedge (a,b) implies bulk removal of all pair edges π_{uv}
  - But, each node in A_u and A_v has different b value
  - Does not map to a clean matching-type problem
- A greedy approach
  - Pick superedges by increasing $|\pi_{uv}|$ value
  - Delete (u,v) if that doesn't violate ϵ-bound for nodes in $A_u \cup A_v$
  - If there is correction (a,b) for π_{uv} in C, we cannot remove (u,v); since removing (u,v) violates error bound for a or b



APXMDL

- Compute the R(S,C) for G
- Find C_ϵ
  - Compute H, with V_H=C
  - Find maximum b-matching M for H; C_ϵ=C-M
- Find S_ϵ
  - Pick superedges (u,v) in S having no correction in C_ϵ in increasing $|\pi_{uv}|$ value
  - Remove (u,v) if that doesn't violate ϵ-bound for any node in $A_u \cup A_v$
- Axp-representation $R_\epsilon=(C_\epsilon, S_\epsilon)$



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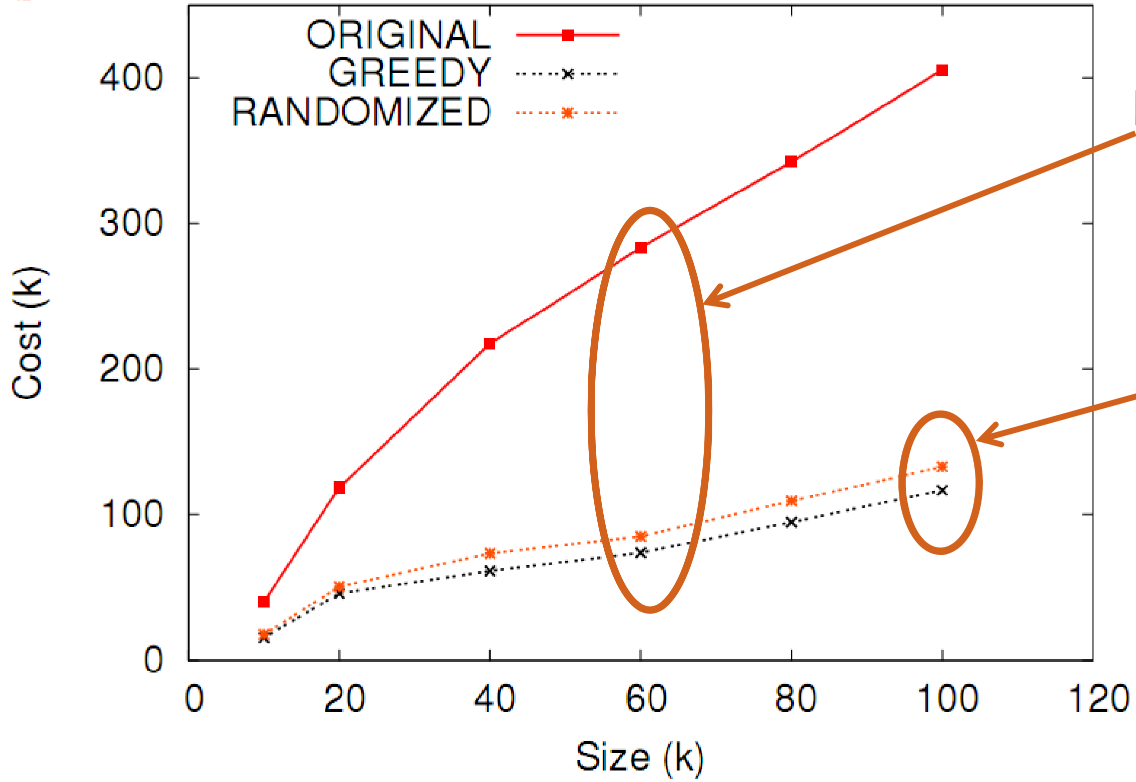
Experimental set-up

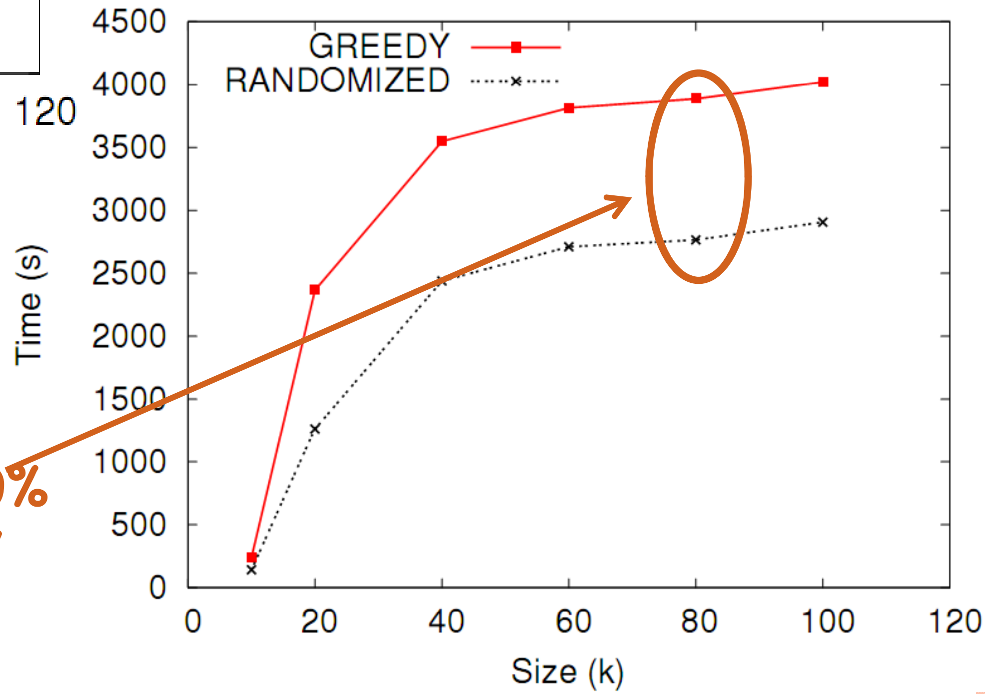
- Algorithms to compare
  - Our techniques GREEDY, RANDOMIZED, APXMDL
  - REF: reference encoding used for web-graph compression
    (we disabled bit-level encoding techniques)
  - GRAC: graph clustering algorithm
    (make supernodes for clusters returned)
- Datasets
  - CNR: web-graph dataset
  - Routeview: autonomous systems topology of the internet
  - Wordnet: English words, edges between related words (synonym, similar, etc.)
  - Facebook: social networking

Cost Reduction (CNR dataset)



Reduces the cost down to 40%

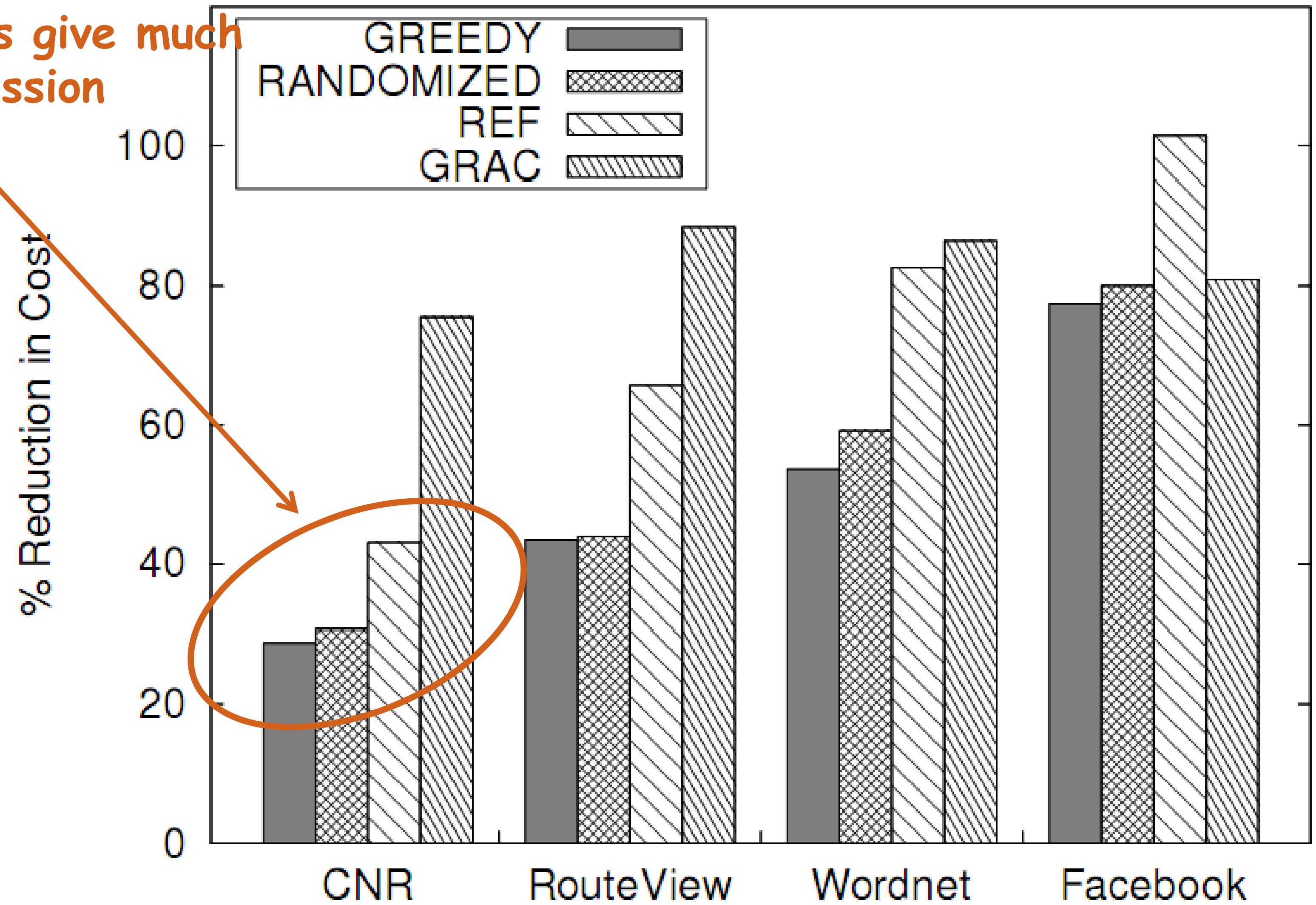
Cost of GREEDY 20% lower than RANDOMIZED



RANDOMIZED is 60% faster than GREEDY

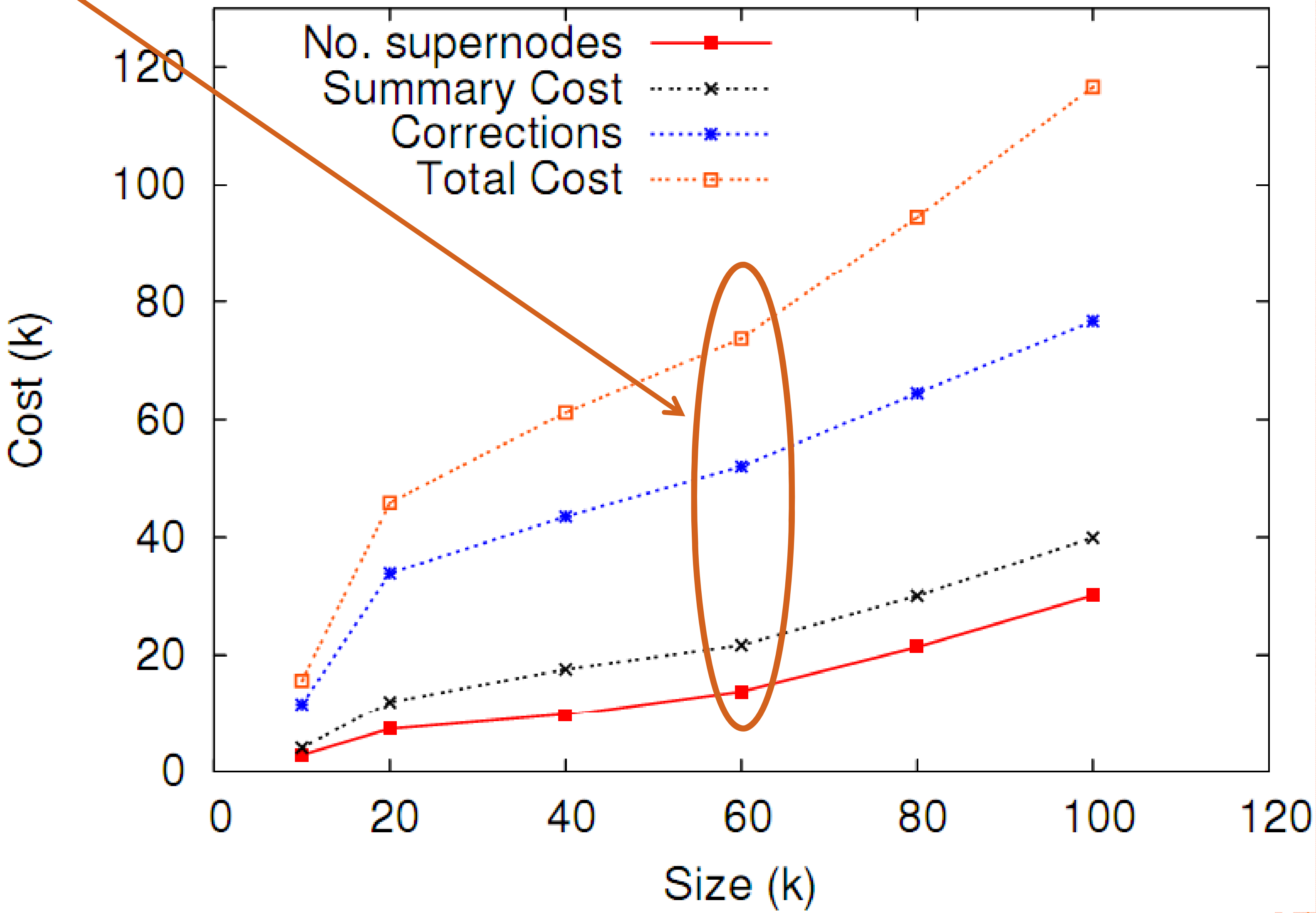
Comparison with other schemes

Our techniques give much better compression

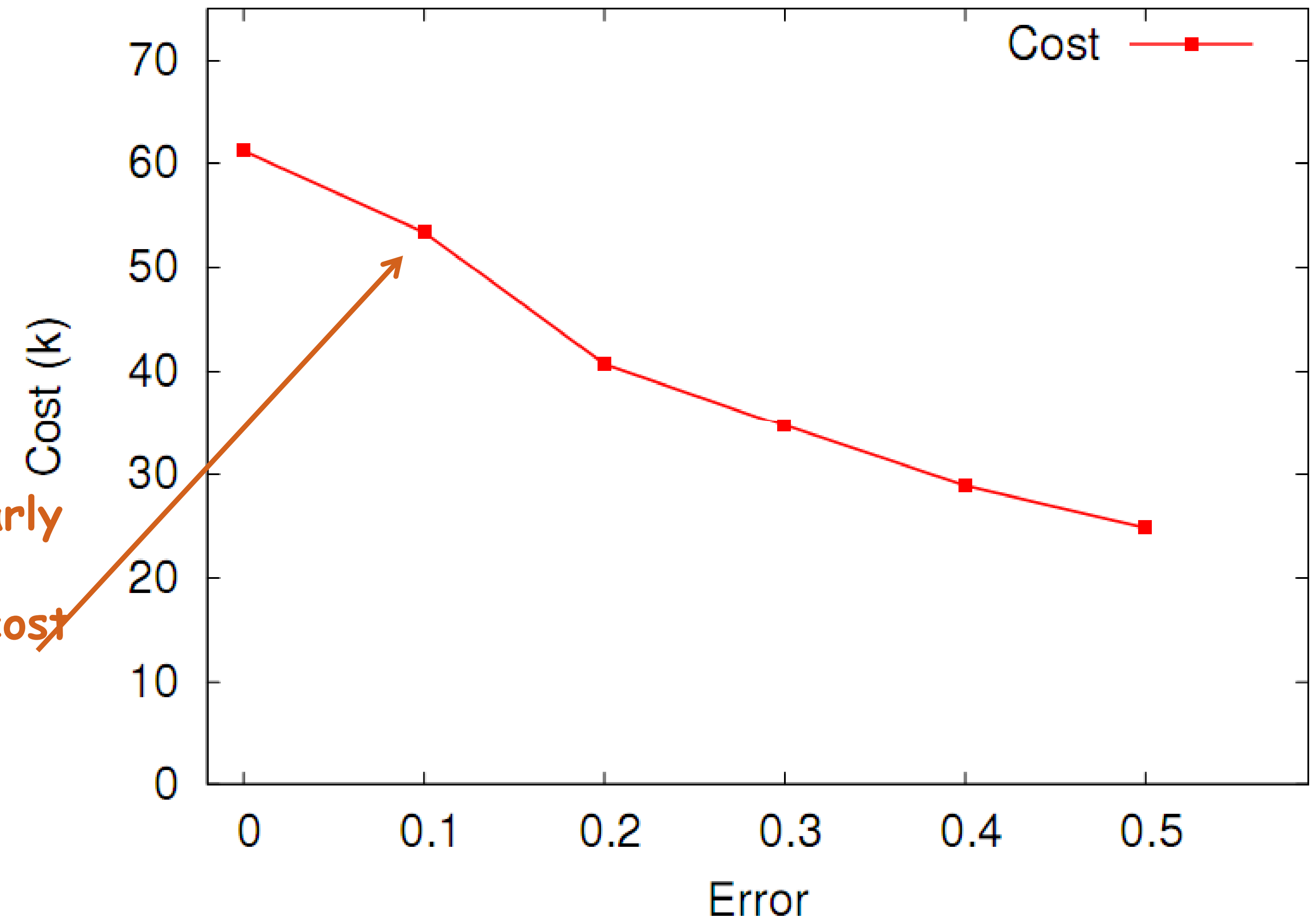


Cost Breakup (CNR dataset)

80% cost of representation is due to corrections



APX-REPRESENTATION



Cost reduces linearly as ϵ is increased; With ϵ=.1, 10% cost reduction over R

Conclusions

- MDL-based representation R(S,C) for graphs
  - Compact summary S: highlights trends
  - Corrections C: reconstructs graph together with S
  - Extend to approximate representation with bounded error
  - Our techniques, GREEDY, RANDOMIZED give up to 40% cost reduction

- Future directions
  - Hardness of finding minimum-cost representation
  - Running graph algorithms (approximately) directly on the compressed structure: apx-shortest path with bounded error on S?
  - Extend to labeled/weighted edges

On Compressing Social Networks

- Flavio Chierichetti, University of Rome
- Ravi Kumar, Yahoo! Research
- Silvio Lattanzi, University of Rome
- Michael Mitzenmacher, Harvard
- Alessandro Panconesi, University of Rome
- Prabhakar Raghavan, Yahoo! Research

Behavioural graphs

- Web graphs
- Host graphs
- Social networks
- Collaboration networks
- Sensor networks
- Biological networks
- …

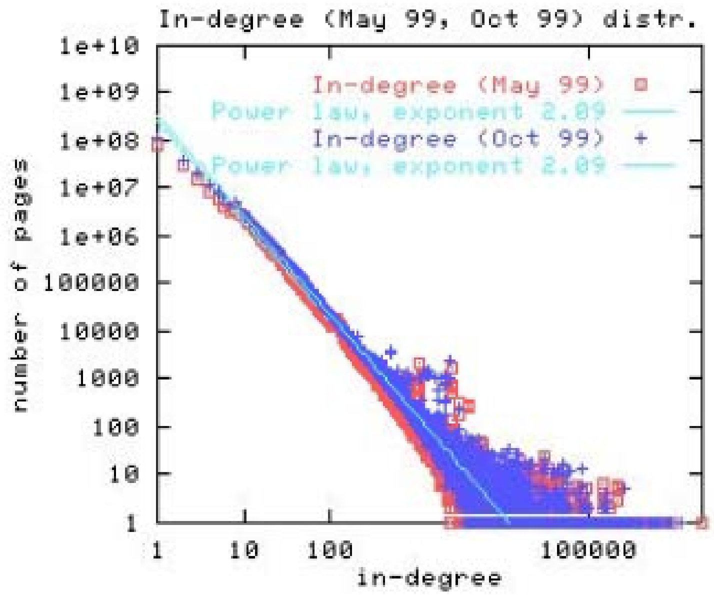
Research trends

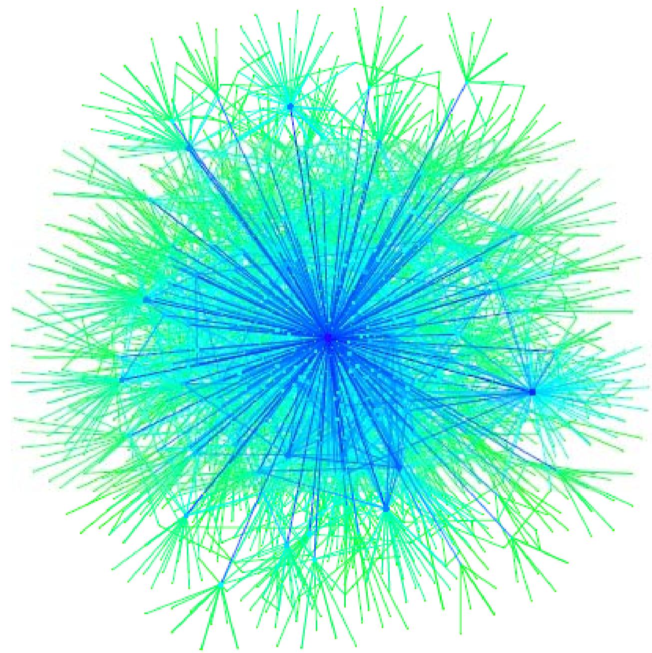
- Empirical analysis: examining properties of real-world graphs
- Modeling: finding good models for behavioural graphs

There has been a tendency to lump together behavioural graphs arising from a variety of contexts

PROPERTIES OF BEHAVIOURAL GRAPHS

- Power law degree distribution
  - Heavy tail
- Clustering
  - High clustering coefficient
- Communities and dense subgraphs
  - Abundance; locally dense, globally sparse; spectrum
- Connectivity
  - Exhibit a “bow-tie” structure; low diameter; small-world phenomenon: Any two vertices are connected by a short path. Two vertices having a common neighbor are more likely to be neighbors.





A REMARKABLE EMPIRICAL FACT

- Snapshots of the web graph can be compressed using less then 3 bits per edge
  - Boldi, Vigna WWW 2004
- Improved to ~2 bits using another data mining inspired compression technique
  - Buehrer, Chellapilla WSDM 2008
- More recent improvements
  - Boldi, Santinin, Vigna WAW 2009

18.5 Mpages, 300 Mlinks from .uk									
R	Average reference chain			Bits/node			Bits/link		
	$W=1$	$W=3$	$W=7$	$W=1$	$W=3$	$W=7$	$W=1$	$W=3$	$W=7$
∞	171.45	198.68	195.98	44.22	38.28	35.81	2.75	2.38	2.22
3	1.04	1.41	1.70	62.31	52.37	48.30	3.87	3.25	3.00
1	0.36	0.55	0.64	81.24	62.96	55.69	5.05	3.91	3.46
Tranpose									
∞	18.50	25.34	26.61	36.23	33.48	31.88	2.25	2.08	1.98
3	0.69	1.01	1.23	37.68	35.09	33.81	2.34	2.18	2.10
1	0.27	0.43	0.51	39.83	36.97	35.69	2.47	2.30	2.22
118 Mpages, 1 Glinks from WebBase									
R	Average reference chain			Bits/node			Bits/link		
	$W=1$	$W=3$	$W=7$	$W=1$	$W=3$	$W=7$	$W=1$	$W=3$	$W=7$
∞	85.27	118.56	119.65	30.99	27.79	26.57	3.59	3.22	3.08
3	0.79	1.10	1.32	38.46	33.86	32.29	4.46	3.92	3.74
1	0.28	0.43	0.51	46.63	38.80	36.02	5.40	4.49	4.17
Tranpose									
∞	27.49	30.69	31.60	27.86	25.97	24.96	3.23	3.01	2.89
3	0.76	1.09	1.31	29.20	27.40	26.75	3.38	3.17	3.10
1	0.29	0.46	0.54	31.09	29.00	28.35	3.60	3.36	3.28

Key insights

1. Many web pages have similar set of neighbors
2. Edges tend to be "local"

ARE SOCIAL NETWORKS COMPRESSIBLE?

- Review of BV compression
- A different compression mechanism that works better for social networks
- A heuristic
- its performance
- and a formalization
- Why study this question?
  - Efficient storage
    - Serve adjacency queries efficiently in-memory
    - Archival purposes – **multiple snapshots**
  - Obtain insights
    - Compression has to utilize special structure of the network
    - Study the randomness in such networks

Adjacency table representation

- Each row corresponds to a node u in the graph
- Entries in a row are sorted integers, representing the neighborhood of u, i.e., edges (u, v)
  - 1: 1, 2, 4, 8, 16, 32, 64
  - 2: 1, 4, 9, 16, 25, 36, 49, 64
  - 3: 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144
  - 4: 1, 4, 8, 16, 25, 36, 49, 64
- Can answer adjacency queries fast
- Expensive (better than storing a list of edges)

Boldi-Vigna (BV): Main ideas

1: 1, 2, 4, 8, 16, 32, 64
2: 1, 4, 9, 16, 25, 36, 49, 64
3: 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144
4: 1, 4, 8, 16, 25, 36, 49, 64

- Similar neighborhoods: The neighborhood of a web page can be expressed in terms of other web pages with similar neighborhoods
  - Rows in adjacency table have similar entries
  - Possible to choose to *prototype* row
- Locality: Most edges are intra-host and hence local
  - Small integers can represent edge destination wrt source
- Gap encoding: Instead of storing destination of each edge, store the difference from the previous entry in the same row

Finding similar neighborhoods

- Canonical ordering: Sort URLs lexicographically, treating them as strings

```
...
17: www.stanford.edu/alchemy
18: www.stanford.edu/biology
19: www.stanford.edu/biology/plant
20: www.stanford.edu/biology/plant/copyright
21: www.stanford.edu/biology/plant/people
22: www.stanford.edu/chemistry
...
```

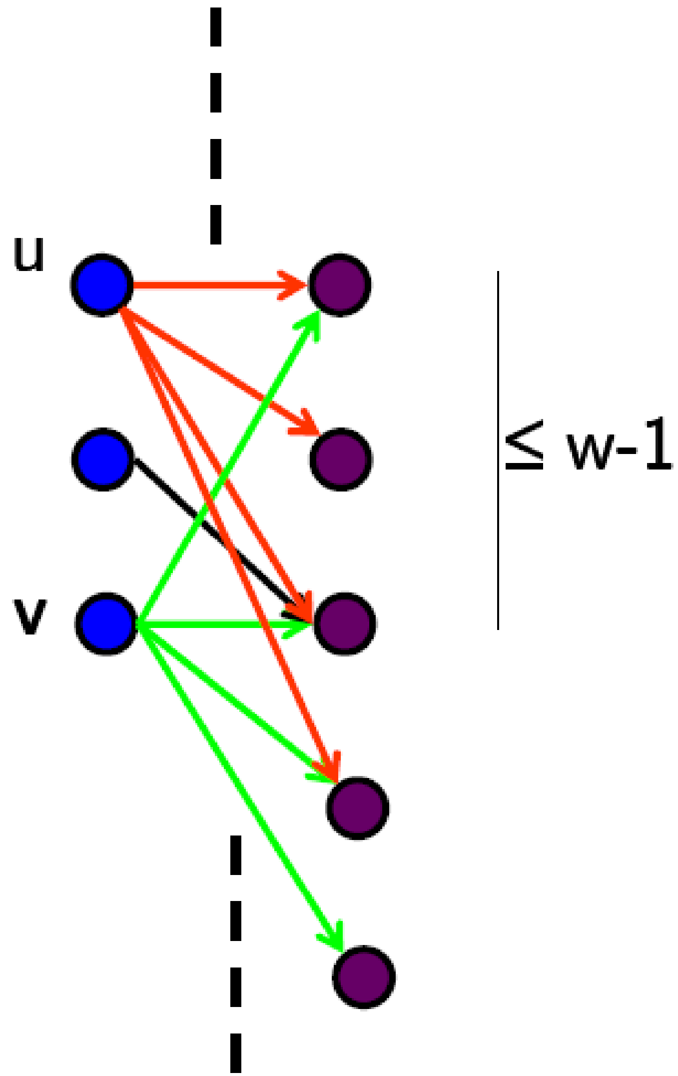
- This gives an identifier for each URL
  Source and destination of edges are likely to get nearby IDs
  - Templated webpages
  - Many edges are intra-host or intra-site

Gap encodings

- Given a sorted list of integers x, y, z, …, **represent** them by x, y-x, z-y, …
- Compress each integer using a code
  - γ code: x is represented by concatenation of unary representation of $\lfloor \lg x \rfloor$ (length of x in bits) followed by binary representation of $x - 2^{\lfloor \lg x \rfloor}$
    Number of bits = $1 + 2\lfloor \lg x \rfloor$
    (see slide 12, http://vigna.dsi.unimi.it/algoweb/webgraph.pdf)
  - δ code: …
  - Information theoretic bound: $1 + \lfloor \lg x \rfloor$ bits
  - ζ code: Works well for integers from a power law
    Boldi Vigna DCC 2004

BV compression

- Each node has a unique ID from the canonical ordering
- Let w = copying window parameter
- To encode a node v
  - Check if out-neighbors of v are similar to any of w-1 previous nodes in the ordering
  - If yes, let u be the prototype: use lg w bits to encode the gap from v to u + difference between out-neighbors of u and v
  - If no, write lg w zeros and encode out-neighbors of v explicitly
- Use gap encoding on top of this



MAIN ADVANTAGES OF BV

- Depends only on locality in a canonical ordering
  - Lexicographic ordering works well for web graph
- Adjacency queries can be answered very efficiently
  - To fetch out-neighbors, trace back the chain of prototypes until a list whose encoding beings with lg w zeros is obtained (no-prototype case)
  - This chain is typically short in practice (since similarity is mostly intra-host)
  - Can also explicitly limit the length of the chain during encoding
- Easy to implement and a one-pass algorithm

BACKLINKS (BL) COMPRESSION

- Social networks are highly *reciprocal*, despite being directed
  - If A is a friend of B, then it is likely B is also A's friend
- (u, v) is reciprocal if (v, u) also exists reciprocal(u) = set of v's such that (u, v) is reciprocal
- How to exploit reciprocity in compression?
  - Can avoid storing reciprocal edges twice
  - Just the reciprocity "bit" is sufficient

Backlinks compression (contd)

- Given a canonical ordering of nodes and copying window w
- To encode a node v
  - Base information: encode out-degree of v minus 1 (if self loop) minus #reciprocal(v) + “self-loop” bit
  - Try to choose a prototype u as in BV within a window w
  - If yes, encode the difference between out-neighbors of u and non-reciprocal out-neighbors of v
    - Encode the gap between u and v
    - Specify which out-neighbors of u are present in v
    - For the rest of out-neighbors of v, encode them as gaps
  - Encode the reciprocal out-neighbors of v
    - For each out-neighbor v’ of v and v’ > v, store if v’ $\in$ reciprocal(v) or not; discard the edge (v’, v)

Canonical orderings

- BV and BL compressions depend just on obtaining a canonical ordering of nodes
  - This canonical ordering should exploit neighborhood similarity and edge locality
- Question: how to obtain a good canonical ordering?
  - Unlike the web page case, it is unclear if social networks have a natural canonical ordering
- Caveat: BV/BL is only one genre of compression scheme
  - Lack of good canonical ordering does not mean graph is incompressible

Some canonical orderings in behavioral graphs

- Random order
- Natural order
  - Time of joining in a social network
  - Lexicographic order of URLs
  - Crawl order
- Graph traversal orders
  - BFS and DFS
- Geographic location: order by zip codes
  - Produces a bucket order
- Ties can be broken using more than one order

Performance of simple orderings

Graph	#nodes	#edges	%reciprocal edges
Flickr	25.1M	69.7M	64.4
UK host graph	0.58M	12.8M	18.6
IndoChina	7.4M	194.1M	20.9

BV

Graph	Natural	Random	DFS
Flickr	21.8	23.9	22.9
UK host	10.8	15.5	14.6
IndoChina	2.02	21.44	-

BL

Graph	Natural	Random	DFS
Flickr	16.4	17.8	17.2
UK host	10.5	14.5	13.8
IndoChina	2.35	17.6	-

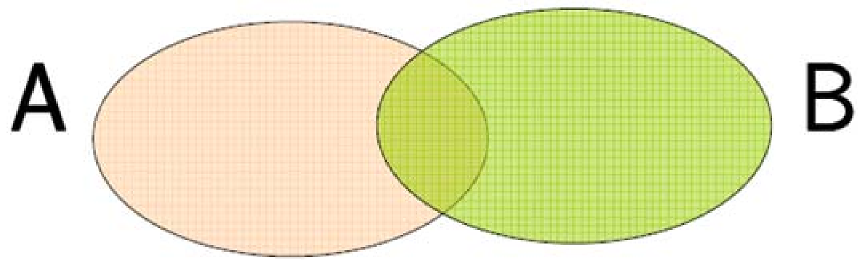
Shingle ordering heuristic

- Obtain a canonical ordering by bringing nodes with similar neighborhoods close together
- Fingerprint neighborhood of each node and order the nodes according to the fingerprint
  - If fingerprint can capture neighborhood similarity and edge locality, then it will produce good compression via BV/BL, provided the graph has amenable
- Use Jaccard coefficient to measure similarity between nodes

$$J(A, B) = |A \cap B| / |A \cup B|$$

A FINGERPRINT FOR JACCARD

- Fingerprint to measure set overlap
  - Shingles have since seen wide usage to estimate the similarity of web pages using a particular feature extraction scheme based on overlapping windows of terms (motivating the name "shingles")



$$M_\pi(A) = \pi^{-1}(\min_{a \in A} \{ \pi(a) \})$$
$$Pr_\pi [M_\pi(A) = M_\pi(B)] = |A \cap B| / |A \cup B|$$

  - The probability that the smallest element of A and B is the same, where smallest is defined by the permutation π , is exactly the similarity of the two sets according to the Jaccard coefficient.
- Min-wise independent permutations suffice
  Broder, Charikar, Frieze, Mitzenmacher STOC 1998
- Hash functions work well in practice

Shingle ordering heuristic (contd)

- Fingerprint of a node $u = M_{\pi}(\text{out-neighbors of } u)$
- Order the nodes by their fingerprint
  - Two nodes with lot of overlapping neighbors are likely to have same shingle
- Double shingle order: break ties within shingle order using a second shingle

Performance of shingle ordering

BV

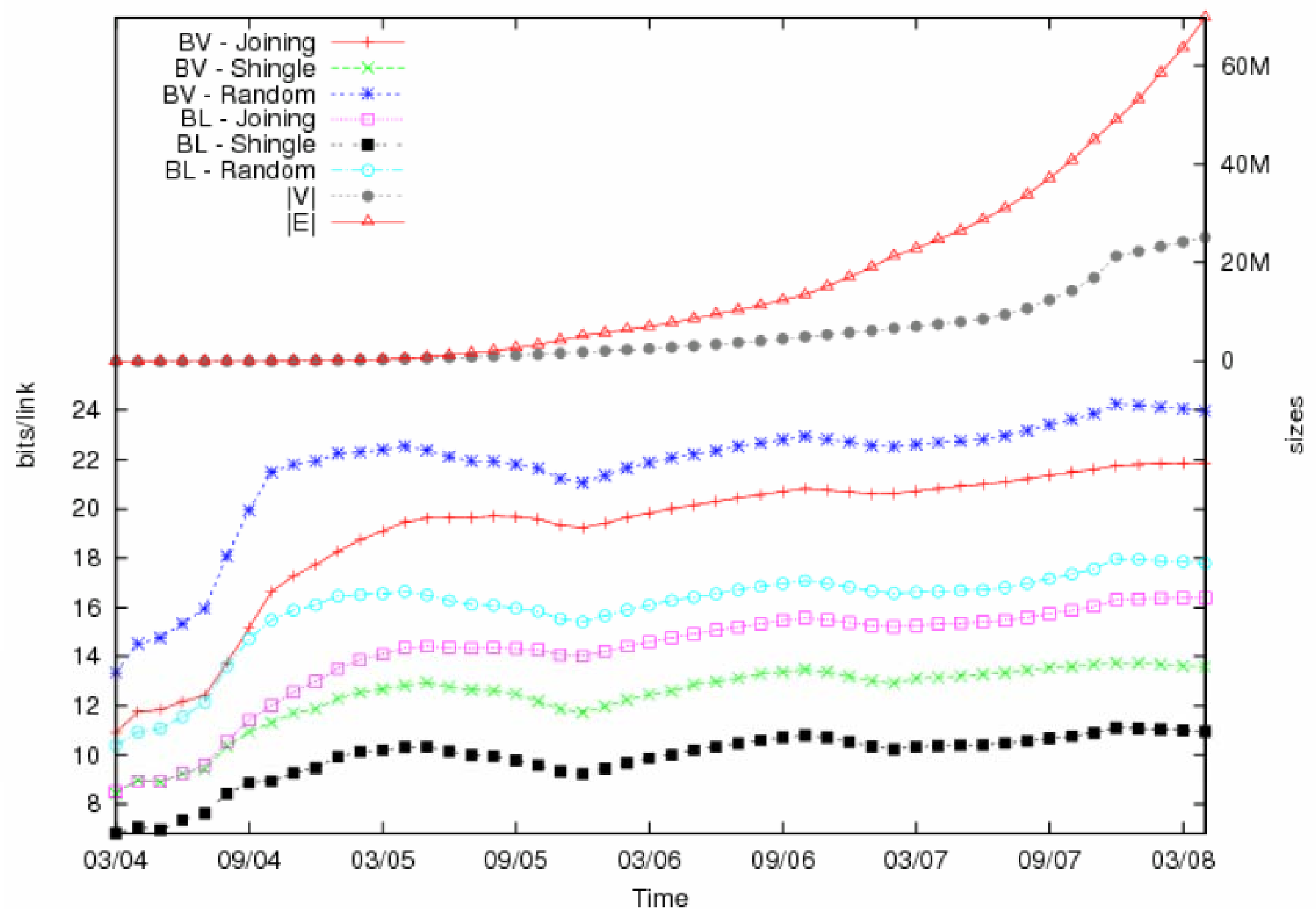
Graph	Natural	Shingle	Double shingle
Flickr	21.8	13.5	13.5
UK host	10.8	8.2	8.1
IndoChina	2.02	2.7	2.7

BL

Graph	Natural	Shingle	Double shingle
Flickr	16.4	10.9	10.9
UK host	10.5	8.2	8.1
IndoChina	2.35	2.7	2.7

Geography does not seem to help for Flickr graph

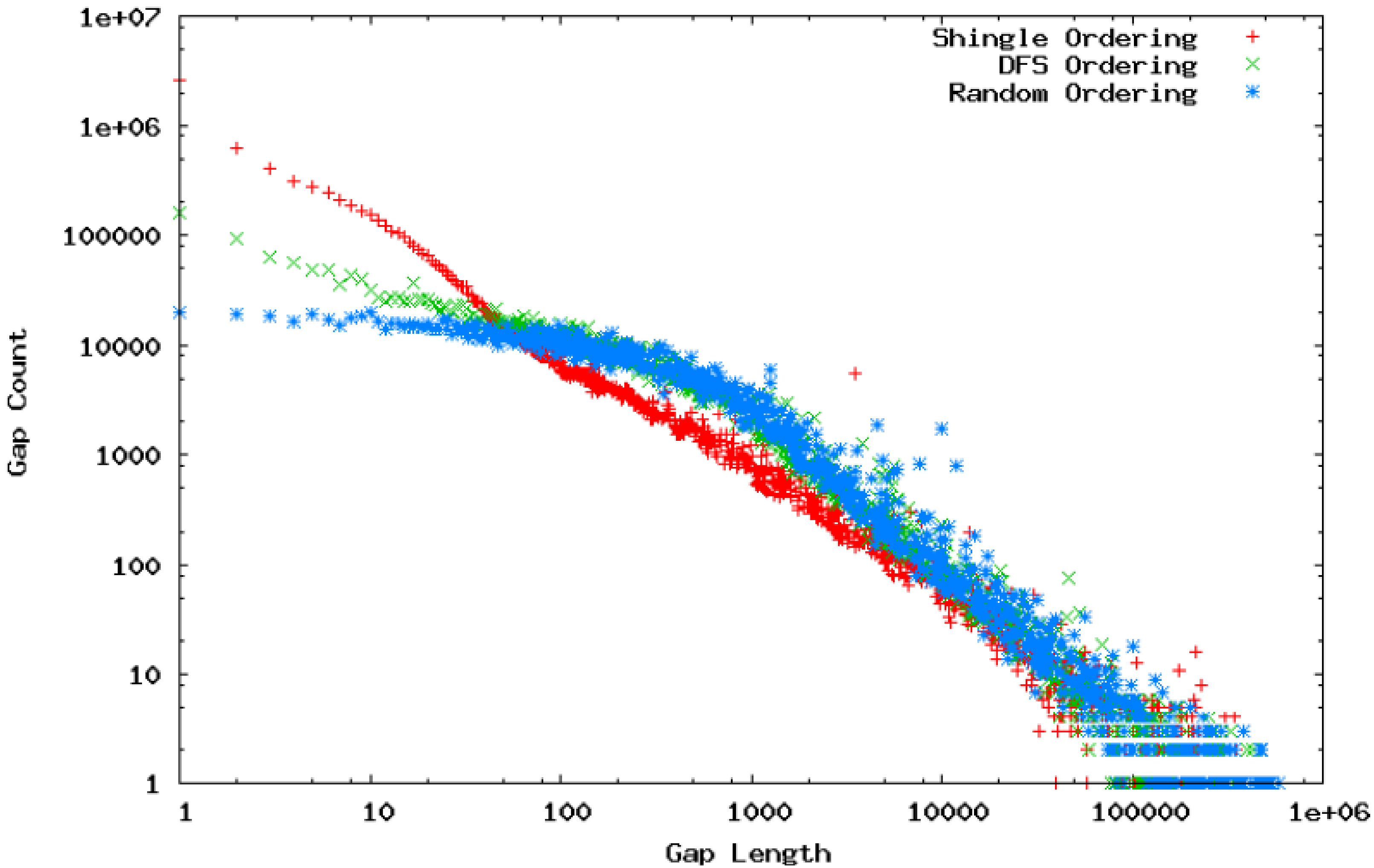
Flickr: Compressibility over time



A PROPERTY OF SHINGLE ORDERING

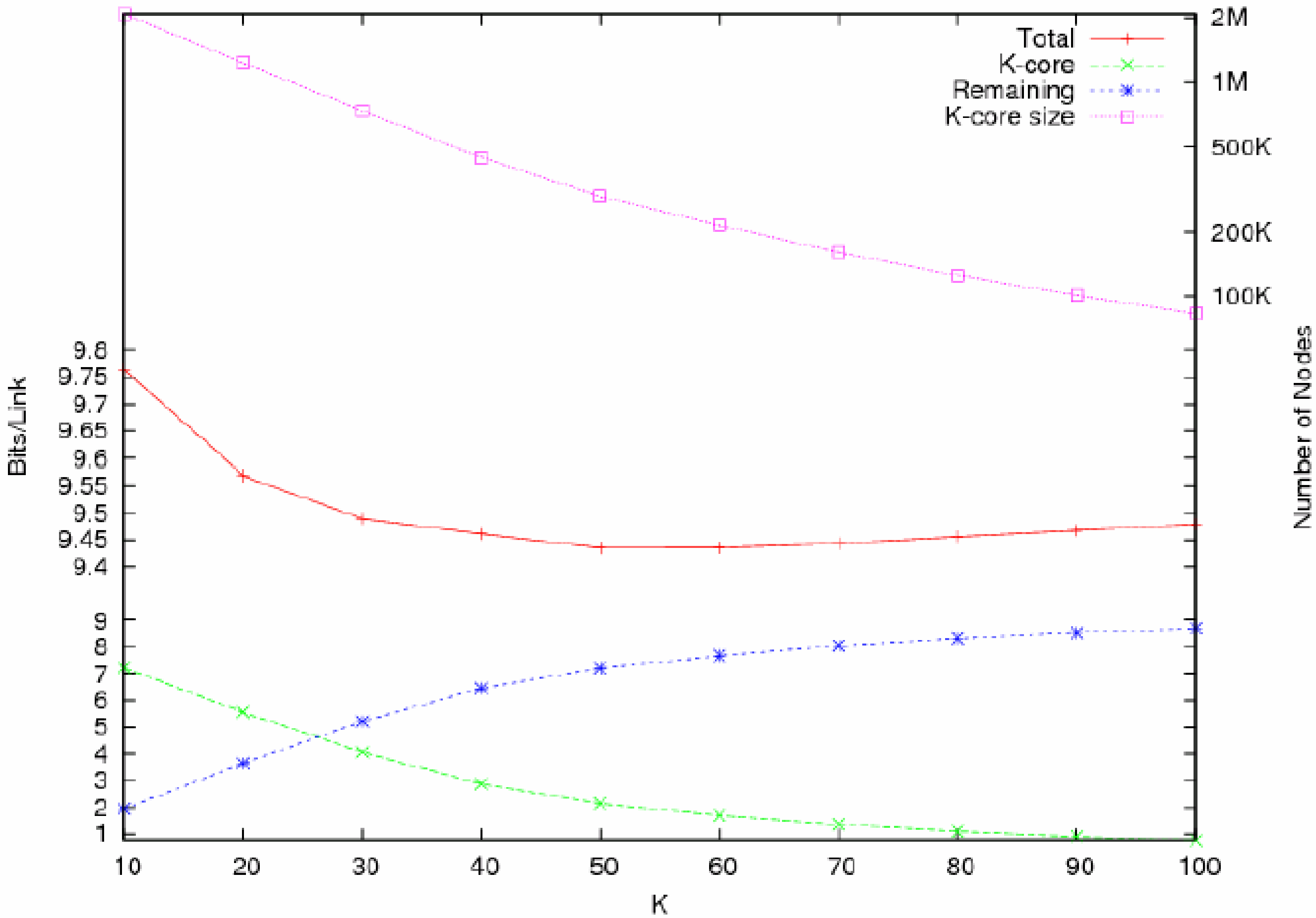
- *Theorem*. Using shingle ordering, a constant fraction of edges will be “copied” in graphs generated by preferential attachment/copying models
- Preferential attachment model: Rich get richer – a new node links to an existing node with probability proportional to its degree
- Shows that shingle ordering helps BV/BL-style compressions in stylized graph models

Gap distribution



Shingle ordering produces smaller gaps

WHO IS THE CULPRIT



Low degree nodes are responsible for incompressibility

Compression-friendly orderings

- In BV/BL, canonical order is all that matters
- Problem. Given a graph, find the canonical ordering that will produce the best compression in BV/BL
  - The ordering should capture locality and similarity
  - The ordering must help BV/BL-style compressions
- We propose two formulations of this problem

MLogA formulation

MLogA. Find an ordering p of nodes such that

$$\Sigma_{(u,v) \in E} \lg |\pi(u)-\pi(v)|$$

is minimized

- Minimize sum of encoding gaps of edges
- Without lg, this is min linear arrangement (MLinA)
- MLinA is well-studied ((log n) log log n) approximable, …
- MLinA and MLogA are very different problems

Theorem. MLogA is NP-hard

- Proof using the inapproximability of MaxCut

MLOGGAPA FORMULATION

- MLogGapA. For an ordering p, let $f_\pi(u)$ = cost of compressing the out-neighbors of u under π
- If $u_1, \ldots, u_k$ are out-neighbors ordered wrt π, $u_0 = u$

$$f_\pi(u) = \sum_{i=1..k} \lg |\pi(u_i)-\pi(u_{i-1})|$$

- Find an ordering π of nodes to minimize $\sum_u f_\pi(u)$
- Minimize encoding gaps of neighbors of a node
- MLogGapA and MLogA are very different problems
  - *Theorem*. MLinGapA is NP-hard
  - *Conjecture*. MLogGapA is NP-hard

Summary

- Social networks appear to be not very compressible
- Host graphs are equally challenging

- These two graphs are very unlike the web graph, which is highly compressible

- Future directions
  - Can we compress social networks better? *Boldi, Santini, Vigna 2009*
  - Is there a lower bound on incompressibility? Our analysis applies only to BV-style compressions
  - Algorithmic questions: Hardness of MLogGapA, Good approximation algorithms
  - Modeling: Compressibility of existing graph models, More nuanced models for the compressible web *Chierichetti, Kumar, Lattanzi, Mitzenmacher, Panconesi, Raghavan FOCS 2009*

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The End

- Thank You