SEG 5520: Optimization I	2007–08 Second Term

Homework Set 1

Instructor: Anthony Man–Cho So — Due: January 22, 2008

SOLVE THE FOLLOWING PROBLEMS:

Problem 1 (15pts). Consider the following problem:

$$\begin{array}{ll} \text{minimize} & \dfrac{c^T x + d}{f^T x + g} \\ \text{subject to} & Ax \le b \\ & f^T x + g \ge 0 \end{array}$$

Here, we assume that $a/0 = +\infty$ if $a > 0$, and $a/0 = -\infty$ if $a \le 0$. Give an equivalent linear programming formulation of the above problem.

Problem 2 (20pts). Let $P = \left\{ x \in \mathbb{R}^n : a_i^T x \le b_i \text{ for } i = 1, \ldots, m \right\}$. Recall that a ball B with center y and radius r is defined as the set $B = \{ x \in \mathbb{R}^n : \|x - y\|_2 \le r \}$. We are interested in finding a ball with the largest possible radius, subject to the condition that it is entirely contained within the set P (also known as the *largest inscribed ball* in P). Give a linear programming formulation of this problem.

Problem 3 (25pts). Let $S = \left\{ x \in \mathbb{R}^n : x^T A x + b^T x + c \le 0 \right\}$, where A is a symmetric $n \times n$ matrix, $b \in \mathbb{R}^n$ and $c \in \mathbb{R}$.

(a) Show that S is convex if $A \succeq \mathbf{0}$.

(b) Let $H = \left\{ x \in \mathbb{R}^n : g^T x + h = 0 \right\}$, where $g \in \mathbb{R}^n \backslash \{\mathbf{0}\}$ and $h \in \mathbb{R}$. Show that $S \cap H$ is convex if $A + \lambda g g^T \succeq \mathbf{0}$ for some $\lambda \in \mathbb{R}$.

Problem 4 (20pts). Let S be a non–empty convex subset of $\mathbb{R}^n$. Let $f : S \to \mathbb{R}$ be a convex function. Prove that if f attains its maximum at a point $\bar{x} \in \operatorname{int} S$, then f must be a constant function.

Problem 5 (20pts). Let $f : \mathbb{R}^n_{++} \to \mathbb{R}$ be a given convex function. Show that the function $g : \mathbb{R}^{n+1}_{++} \to \mathbb{R}$ given by $g(\tau; x) = \tau \cdot f(x/\tau)$ (called the *homogenized version* of f) is also a convex function in the domain of $(\tau; x) \in \mathbb{R}^{n+1}_{++}$.