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MR1696772 (2001d:93004)
Yong, Jiongmin (PRC-FUDAN); **Zhou, Xun Yu** (PRC-CHHK-EMG)
★**Stochastic controls. (English summary)**
Hamiltonian systems and HJB equations.
Applications of Mathematics (New York), 43.
Springer-Verlag, *New York*, 1999. *xxii*+438 *pp*. $79.95. *ISBN* 0-387-98723-1
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Two main tools in the optimal control of dynamic systems are dynamic programming (which leads to a partial differential equation (PDE) for the cost-to-go function, commonly known as the Hamilton-Jacobi-Bellman (HJB) equation), and Pontryagin's maximum principle (which involves Hamiltonian systems). The relationship between these two approaches is well established in the case of deterministic systems, and can be found in standard texts on optimal control. For stochastic systems, however, the relationship was not as well established until recently. Until about twenty years ago, even the stochastic maximum principle was not established rigorously for the most general case where the control also affects the diffusion term and the control constraint set is not convex. In the late 1980s the two authors, along with some collaborators at Fudan University (China), studied these questions and developed a comprehensive theory which not only addressed the stochastic maximum principle in its greatest generality, but also established the relationship between the resulting Hamiltonian systems (which involve stochastic ordinary differential equations) and the HJB equation (which is a PDE). These results form the core of this book and are supplemented by related results of other researchers on the topic and by some introductory material to make this a self-contained volume.

  The book is very well written, and will undoubtedly remain a major reference on the topic for years to come. It comprises seven chapters; the first is devoted to introductory material on stochastic calculus, essential for the uninitiated reader to be able to follow the more advanced material contained in the other six chapters. Chapter 2 provides a mathematically precise formulation of the underlying stochastic control problem, illustrated by several realistic examples. This chapter, as well as the next four chapters, starts with the deterministic optimal control problem, which is

useful for the reader to see the main differences (as well as similarities) between deterministic and stochastic formulations and the ensuing theories for the two cases.

Chapter 3 deals with the stochastic maximum principle and provides a rigorous derivation of the associated stochastic Hamiltonian system which consists of two backward stochastic differential equations (SDEs) (adjoint equations) and one forward SDE (the original state dynamics), along with the condition of maximization of a Hamiltonian. Sufficiency of the maximum principle is also discussed in this chapter. Chapter 4, on the other hand, is devoted to the other tool of optimization—dynamic programming. The authors allow the value function to be nonsmooth, and hence work with viscosity solutions of the HJB equation. They provide a proof of the principle of optimality using the weak formulation, and a simplified proof of the uniqueness of the viscosity solution.

Chapter 5 is on the relationship between the two tools introduced in the previous two chapters. It first reviews the classical Hamilton-Jacobi theory in mechanics, to demonstrate the origin of the study of this relationship. Then, the relationship is investigated for deterministic systems and is compared with the methods of characteristics in PDE theory, the Feynman-Kac formula in probability theory, and the shadow price in economics. After completing this background material for deterministic systems, the authors turn to the stochastic case, first discussing the case of smooth value functions, and then providing a detailed analysis of the nonsmooth case. The chapter also includes stochastic verification theorems for the nonsmooth case, and some discussion on the construction of optimal feedback controls.

Chapter 6 specializes the theories of the previous chapters to the linear quadratic case, where the system dynamics are linear in the state and the control, and the cost function is quadratic in both, with the control weighting matrix allowed to be indefinite. Both the maximum principle and the principle of optimality lead to the same stochastic Riccati equation in this case, in terms of which the optimal feedback control is constructed. The chapter ends with a specific example, involving mean-variance portfolio selection.

Finally, Chapter 7 presents the results of more recent research on backward and forward-backward SDEs, with an emphasis on the relationship between nonlinear SDEs and nonlinear PDEs. The results should also be of independent interest, going beyond the application area of stochastic control that motivated their study. The chapter begins with the original arguments of Bismut for studying linear backward SDEs by using the martingale representation theorem. Following this, the existence and uniqueness of solutions to nonlinear backward SDEs are investigated for two types of time durations: finite deterministic and random horizon. This is followed by presentation of Feynman-Kac-type formulae with respect to both forward and backward SDEs. Next, a kind of inverse of the Feynman-Kac-type formulae, the so-called four-step scheme, is discussed; this enables solutions to forward-backward SDEs to be represented by those to PDEs. Solvability and nonsolvability of forward-backward SDEs are also analyzed in this chapter. Finally, the four-step scheme is used to derive the Black-Scholes formula in option pricing.

This is an authoritative book which should be of interest to researchers in stochastic control, mathematical finance, probability theory, and applied mathematics. Material out of this book could also be used in graduate courses on stochastic control and dynamic optimization in mathematics,

engineering, and finance curricula.

Reviewed by *Tamer Başar*